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LETTER TO THE EDITORS

A COMMENT ON THE PAPER "AN EFFECT OF A TRANSVERSE TEMPERATURE GRADIENT ON TURBULENT PIPE FLOW" BY W. MURGATROYD

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IT CAN BE ARGUED very simply on physical grounds that the suggested correction term [1] to the turbulent shear stress cannot exist. The Navier–Stokes equation for compressible flow in the x-direction of a two dimensional x, y system can be written,

$$\frac{\partial p}{\partial x} = \frac{\mu\, \partial^2 \bar{U}}{\partial y^2} - \frac{\partial\, \overline{\rho V u}}{\partial y} - \overline{\rho V}\, \frac{\partial \bar{U}}{\partial y} - \bar{U}\, \frac{\partial \overline{\rho V}}{\partial y} \qquad (10)$$

The terms on the right-hand side are the first derivatives of the shear stresses arising respectively from viscosity, turbulent fluctuations, mass transfer in the y-direction and the first derivative of mass transfer in the y-direction.

The term $\bar{\rho}\bar{U}_j(\partial \bar{U}_i/\partial x_i)$ in equation (1) should be $(\partial/\partial x_i)(\bar{\rho}\bar{U}_j\bar{U}_i)$. Equation (1) can then be seen to reduce to equation (10).

For the case considered, flow between infinite parallel plates, there can be no net mass transfer in the y-direction, whether there is a temperature gradient or not, i.e.

$$\overline{\rho V} = \bar{\rho}\bar{V} + \overline{\rho v} = 0. \qquad (11)$$

This immediately gives the conclusion that only the usual turbulent shear stress and viscous shear stress operate, whether there is a temperature gradient or not.

It can be seen from equation (11) that if there is a temperature gradient giving, as is shown in the note, a finite value to $\overline{\rho v}$ then the assumption $\bar{V} = 0$ is not valid.

Substituting (11) in (10) and expanding

$$\frac{\partial p}{\partial x} = \frac{\mu\, \partial^2 \bar{U}}{\partial y^2} - \frac{\partial}{\partial y}\left(\bar{\rho}\, \overline{uv} + \frac{\overline{\rho u}\,\overline{\rho v}}{\bar{\rho}} + \overline{\rho u v}\right).$$

The last two terms in the bracket are clearly very much smaller than the first term, giving the usual expression for the eddy diffusivity of momentum. The correction suggested in the note arises from a combination of an, error in the basic equation and an invalid assumption $\bar{V} = 0$.

REFERENCE

1. W. MURGATROYD, *Int. J. Heat Mass Transfer* **8**, 857 (1965).

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